Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 9 3 7 7 2 5 5 2 2 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find, in terms of a, the set of values of x satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where a is a positive constant. [4]

2 Solve the equation $\cos(\theta - 60°) = 3 \sin \theta$, for $0° \leqslant \theta \leqslant 360°$. [5]

3 **(a)** Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x. [3]

(b) Hence solve the equation $\log_3(4y + 1) = 1 + 2\log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

4 The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b \sin 2x)e^{-4x}$, where a and b are constants. [4]

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(b) Hence find the exact x-coordinates of the two stationary points. [3]

5 The complex number $3 - \mathrm{i}$ is denoted by u.

(a) Show, on an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O, A, B and C. [3]

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(b) Express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(c) By considering the argument of $\frac{u^*}{u}$, or otherwise, prove that $\tan^{-1}\left(\frac{3}{4}\right) = 2\tan^{-1}\left(\frac{1}{3}\right)$. [2]

6 The parametric equations of a curve are $x = \dfrac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{\cos t}{\sin^2 t}$. [5]

(b) Find the equation of the tangent to the curve at the point where $y = 0$. [3]

7 Let $f(x) = \dfrac{5x^2 + 8x - 3}{(x-2)(2x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

(c) Obtain an expression for N in terms of t, and find the greatest value of N predicted by this model. [2]

9 With respect to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line l has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(a) Find in degrees the acute angle between the directions of OA and l. [3]

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(b) Find the position vector of the foot of the perpendicular from A to l. [4]

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(c) Hence find the position vector of the reflection of A in l. [2]

10 The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$. [5]

(b) Verify by calculation that a lies between 2.4 and 2.8. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.