Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5176306371

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The diameters, x millimetres, of a random sample of 200 discs made by a certain machine were recorded. The results are summarised below.

$$n = 200 \qquad \Sigma x = 2520 \qquad \Sigma x^2 = 31\,852$$

(a) Calculate a 95% confidence interval for the population mean diameter. [6]

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(b) Jean chose 40 random samples and used each sample to calculate a 95% confidence interval for the population mean diameter.

How many of these 40 confidence intervals would be expected to include the true value of the population mean diameter? [1]

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2 Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of the die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the correct score on exactly 5 out of 15 throws.

Test Arvind's claim at the 10% significance level. [5]

3 The lengths, in centimetres, of two types of insect, A and B, are modelled by the random variables $X \sim N(6.2, 0.36)$ and $Y \sim N(2.4, 0.25)$ respectively.

Find the probability that the length of a randomly chosen type A insect is greater than the sum of the lengths of 3 randomly chosen type B insects. [5]

4 The independent random variables X and Y have distributions $\mathrm{Po}(2)$ and $\mathrm{B}\left(20, \frac{1}{4}\right)$ respectively.

(a) Find the mean and standard deviation of $X - 3Y$. [5]

(b) Find $P(Y = 15X)$. [3]

5 Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

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(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

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Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

6 A random variable X has probability density function f. The graph of $\mathrm{f}(x)$ is a straight line segment parallel to the x-axis from $x = 0$ to $x = a$, where a is a positive constant.

(a) State, in terms of a, the median of X. [1]

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(b) Find $\mathrm{P}(X > \frac{3}{4}a)$. [1]

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(c) Show that $\mathrm{Var}(X) = \frac{1}{12}a^2$. [5]

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(d) Given that $P(X < b) = p$, where $0 < b < \frac{1}{2}a$, find $P\left(\frac{1}{3}b < X < a - \frac{1}{3}b\right)$ in terms of p. [2]

7 In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypotheses for Jenny's test. [1]

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(b) Find the probability that Jenny will make a Type I error. [3]

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(c) Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.