Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

6217921484

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where a and b are integers. [4]

2 The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$.

(a) Find the value of a. [2]

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(b) When a has this value, solve the inequality $\mathrm{p}(x) < 0$. [4]

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3 The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

4 **(a)** Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

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(b) Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

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5 **(a)** Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where x and y are real and exact. [3]

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(b) On a sketch of an Argand diagram with origin O, show points A and B representing the roots of the equation in part **(a)**. [1]

(c) Find the exact modulus and argument of each root. [3]

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(d) Hence show that the triangle OAB is equilateral. [1]

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6 Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Using a scalar product, find the cosine of angle BAC. [4]

(b) Hence find the area of triangle ABC. Give your answer in a simplified exact form. [4]

7 The variables x and θ satisfy the differential equation

$$x \sin^2 \theta \frac{\mathrm{d}x}{\mathrm{d}\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

(a) Show that $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to θ is $-\operatorname{cosec}^2 \theta$.) [1]

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(b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

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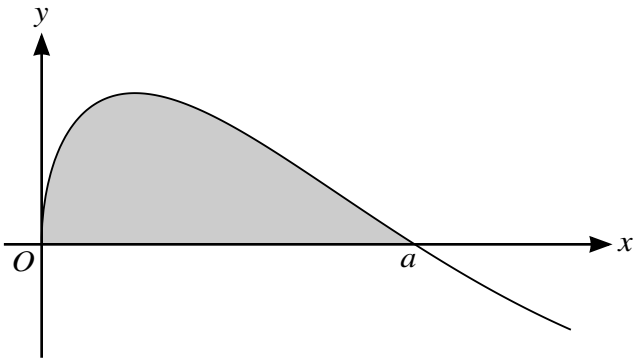

The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the x-axis at the point where $x = a$.

(a) State the exact value of a. [1]

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(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x-axis. [7]

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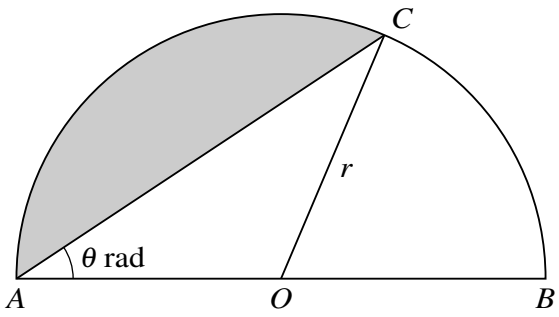

The diagram shows a semicircle with diameter AB, centre O and radius r. The shaded region is the minor segment on the chord AC and its area is one third of the area of the semicircle. The angle CAB is θ radians.

(a) Show that $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$. [4]

(b) Verify by calculation that $0.5 < \theta < 0.7$. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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10 Let $\mathrm{f}(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$. Give your answer as a single logarithm. [5]