



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

2 (a) Sketch the graph of $y = |2x + 3|$.

[1]

(b) Solve the inequality $3x + 8 > |2x + 3|$.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 3 Find the coefficient of x^3 in the binomial expansion of $(3 + x)\sqrt{1 + 4x}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 4 (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0. \quad [2]$$

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ for $0^\circ < \theta < 180^\circ$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

6 Relative to the origin *O*, the points *A*, *B* and *C* have position vectors given by

$$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral *ABCD* is a parallelogram.

(a) Find the position vector of *D*. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The angle between BA and BC is θ .

Find the exact value of $\cos \theta$.

[3]

.....

.....

.....

.....

.....

.....

.....

(c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The variables x and y satisfy the differential equation

$$\cos 2x \frac{dy}{dx} = \frac{4 \tan 2x}{\sin^2 3y},$$

where $0 \leq x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 Let $f(x) = \frac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence find the exact value of $\int_0^4 f(x) dx$, giving your answer in the form $a + b \ln c$, where a , b and c are integers. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9 The constant a is such that $\int_0^a xe^{-2x} \, dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2} \ln(4a + 2)$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Verify by calculation that a lies between 0.5 and 1. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Use an iterative formula based on the equation in (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

10 The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$.

(a) Show that $(x + 3)$ is a factor of $p(x)$. [2]

.....
.....
.....
.....
.....
.....
.....

(b) Show that $z = -1 + 2\sqrt{6}i$ is a root of $p(z) = 0$. [3]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.
