



Cambridge International AS & A Level

CANDIDATE
NAME

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 3 (a) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. [2]

- (b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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4 Solve the equation $2 \cos x - \cos \frac{1}{2}x = 1$ for $0 \leq x \leq 2\pi$. [5]

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5 The complex number $2 + yi$ is denoted by a , where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y . [3]

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(b) Given that $\operatorname{Re}(f(a)) = -20$, find $\arg a$. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α .

(a) Show by calculation that α lies between 0.5 and 1. [2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3} \left(x_n + 4 \tan^{-1} \left(\frac{1}{3x_n} \right) \right)$$

converges, then it converges to α . [2]

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7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x + 2y}{2x + 3y}$. [4]

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- (b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

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8 (a) The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x . [7]

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(b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

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(b) Hence show that $\int_0^1 f(x) \, dx = \frac{5}{2} - \ln 72$. [5]

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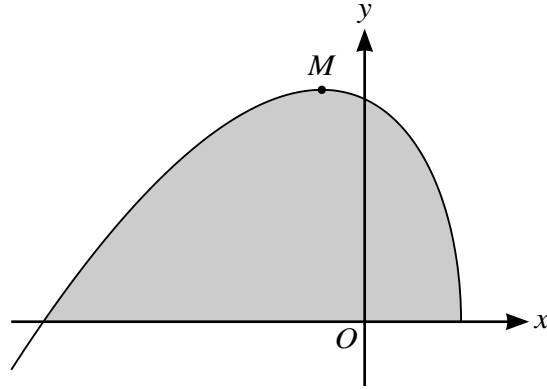
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The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M .

(a) Find the exact coordinates of M .

[5]

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- (b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x -axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

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11 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B . [5]

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