## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


## MATHEMATICS

0580/42
Paper 4 (Extended)
October/November 2023
2 hours 30 minutes

You must answer on the question paper.
You will need: Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For $\pi$, use either your calculator value or 3.142.


## INFORMATION

- The total mark for this paper is 130 .
- The number of marks for each question or part question is shown in brackets [ ].

1


(a) (i) Translate triangle $T$ by the vector $\binom{-7}{1}$. Label the image $K$.
(ii) Describe fully the single transformation that maps triangle $K$ onto triangle $T$.
$\qquad$
$\qquad$
(b) Reflect triangle $T$ in the line $y=4$.
(c) Rotate triangle $T$ through $90^{\circ}$ clockwise about ( 0,0 ).
(d) (i) Enlarge triangle $T$ by scale factor $-\frac{1}{2}$, centre $(0,0)$. Label the image $P$.
(ii) Describe fully the single transformation that maps triangle $P$ onto triangle $T$.
$\qquad$
$\qquad$

2 (a) Daisy records her 50 homework marks.
The table shows the results.

| Homework mark | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 1 | 3 | 19 | 11 | 10 | 6 |

(i) Write down the range.
(ii) Write down the mode.
$\qquad$
(iii) Find the median.
$\qquad$
(iv) Calculate the mean.
$\qquad$
(b) $\begin{array}{llllllllllll}21 & 33 & 20 & 25 & 21 & 34 & 22 & 21 & 20 & 30 & 18\end{array}$

The list shows Ed's scores in 11 tests.
(i) Complete the stem-and-leaf diagram to show this information.

| 1 |  |
| :--- | :--- |
| 2 |  |
| 3 |  |

Key: $2 \mid 5$ represents 25
(ii) Find the median.
(iii) Find the interquartile range.
$\qquad$

3 (a) The value of Priya's car decreases by $10 \%$ every year.
The value today is $\$ 7695$.
(i) Calculate the value of the car after one year.

$$
\$
$$

(ii) Calculate the value of the car one year ago.

> \$
(b) Ali invests $\$ 600$ at a rate of $2 \%$ per year simple interest.

Calculate the value of Ali's investment at the end of 5 years.
\$
(c) Sara invests $\$ 500$ at a rate of $r \%$ per year compound interest.

At the end of 12 years, the value of Sara's investment is $\$ 601.35$, correct to the nearest cent.
Find the value of $r$.
(d) The mass of a radioactive substance decreases exponentially at a rate of $3 \%$ each day.
(i) Find the overall percentage decrease at the end of 10 days.
\% [2]
(ii) Find the number of whole days it takes until the mass of this substance is one half of its original amount.

4 (a)


NOT TO
SCALE

The diagram shows a sector of a circle that is made into a cone by joining $O A$ to $O B$.
The sector angle is $x^{\circ}$ and the radius of the sector is 7.5 cm .
The base radius of the cone is 1.5 cm .
Calculate the value of $x$.

$$
x=
$$

(b)


NOT TO
SCALE

The diagram shows a cylinder with radius 8 cm inside a sphere with radius 17 cm .
Both ends of the cylinder touch the curved surface of the sphere.
(i) Show that the height of the cylinder is 30 cm .
(ii) Calculate the volume of the cylinder as a percentage of the volume of the sphere. [The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]
(c)


NOT TO SCALE

The diagram shows a solid sphere with radius 6 cm inside a cube with side length 20 cm . The cube contains water to a depth of 15 cm . The sphere is removed.

Calculate the new depth of water in the cube.
[The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]

5 (a) In a shop the cost of a fiction book is $\$ x$ and the cost of a reference book is $\$(x+2)$. The cost of 11 fiction books is the same as the cost of 10 reference books.

Find the value of $x$.

$$
x=
$$

(b) In another shop, the cost of a fiction book is $\$ y$ and the cost of a reference book is $\$(y+2)$. Maria spends $\$ 95$ on fiction books and $\$ 147$ on reference books. She buys a total of 12 books.
(i) Show that $6 y^{2}-109 y-95=0$.
(ii) Factorise $6 y^{2}-109 y-95$.
(iii) Find the value of $y$.

$$
\begin{equation*}
y= \tag{1}
\end{equation*}
$$



NOT TO SCALE

The diagram shows a right-angled triangle.
Find the value of $w$.

$$
\begin{equation*}
w= \tag{7}
\end{equation*}
$$

7 (a)


The diagram shows a right-angled triangle $P Q R$ on horizontal ground.
$X$ is vertically above $R$ and the angle of elevation of $X$ from $P$ is $21^{\circ}$. $X R=2.8 \mathrm{~m}$ and $R Q=7.1 \mathrm{~m}$.
(i) Calculate the angle of elevation of $X$ from $Q$.
(ii) Calculate $P Q$.
(b)


Calculate the acute angle $K M L$.
(c)


The area of triangle $A B C$ is $62.89 \mathrm{~cm}^{2}$.
(i) Show that angle $B A C=28.4^{\circ}$, correct to 1 decimal place.
(ii) Calculate $B C$.
(iii) $A B$ is extended to a point $D$ such that angle $B D C=90^{\circ}$.

Calculate $B D$.


Dice A


Dice B

The diagram shows two fair dice.
Dice A is numbered $1,2,2,2,3,6$.
Dice $B$ is numbered $2,3,3,4,4,4$.
(a) (i) Dice A is rolled once.

Write down the probability that it lands on the number 6 .
$\qquad$
(ii) Dice A is rolled 150 times.

Find the number of times it is expected to land on the number 6.
$\qquad$
(b) Dice A and Dice B are each rolled once.
(i) Find the probability that the two numbers they land on have a total of 6 .
(ii) Find the probability that when the two numbers they land on have a total of 6, both numbers are 3.
(c) Dice B is rolled $n$ times.

The probability that on the $n$th roll it first lands on a number 3 is $\frac{32}{729}$.
Find the value of $n$.

$$
n=
$$

9


The diagram shows a sketch of the graph of $y=4 x^{3}-x^{4}$.
The graph crosses the $x$-axis at the origin $O$ and at the point $A$.
The point $B$ is a maximum point.
(a) Differentiate $4 x^{3}-x^{4}$.
$\qquad$
(b) Find the coordinates of $B$.
$\qquad$
(c) Find the gradient of the graph at the point $A$.

10 (a)

$A B C D E F$ is a regular hexagon.
$D F, D A$ and $D B$ are diagonals.
Complete the following statements using three different triangles.
Triangle $D E F$ is congruent to triangle ..................
Triangle $\qquad$ is congruent to triangle $\qquad$
(b)

$P$ and $Q$ are points on the circle with centre $O$.
$T P$ and $T Q$ are tangents to the circle from the point $T$.
Complete the following statements and reasons.
In triangles $O P T$ and $O Q T$
$O P=$ $\qquad$ because each is a radius of the circle
$O T$ is a common side
Angle $O P T=$ angle $\qquad$ $=90^{\circ}$ because $\qquad$
Triangles $O P T$ and $O Q T$ are congruent using the criterion $\qquad$
This proves that the tangents $T P$ and $T Q$ are $\qquad$

11
$\mathrm{f}(x)=1-3 x$
$\mathrm{g}(x)=(x-1)^{2}$
$\mathrm{h}(x)=\frac{3}{x}, x \neq 0$
(a) Find $\mathrm{g}(3)$.
(b) Find $\mathrm{f}(x-2)$, giving your answer in its simplest form.
(c) Find $\mathrm{f}^{-1}(x)$.

$$
\mathrm{f}^{-1}(x)=
$$

(d) $\operatorname{gf}(x)-\mathrm{g}(x) \mathrm{f}(x)=3 x^{3}+a x^{2}+b x+c$

Find the value of each of $a, b$ and $c$.

$$
\begin{align*}
& a= \\
& b= \\
& c= \tag{5}
\end{align*}
$$

(e) Find $\mathrm{h}(x)-\mathrm{f}(x)$, giving your answer as a single fraction in its simplest form.
(f) $\mathrm{h}\left(x^{n}\right)=3 x^{7}$

Find the value of $n$.

12

$O$ is the origin $(0,0), A$ is the point $(8,1)$ and $B$ is the point $(2,5)$.
(a) Write as column vectors.
(i) $\overrightarrow{O B}$

$$
\begin{equation*}
\overrightarrow{O B}=( \tag{1}
\end{equation*}
$$

(ii) $\overrightarrow{A B}$

$$
\overrightarrow{A B}=(\quad)
$$

(b) Find the equation of the line $A B$.

Give your answer in the form $y=m x+c$.
(c) Find the equation of the perpendicular bisector of $A B$.

Give your answer in the form $y=m x+c$.

$$
\begin{equation*}
y= \tag{4}
\end{equation*}
$$

(d) The line $A B$ meets the $y$-axis at $P$.

The perpendicular bisector of $A B$ meets the $y$-axis at $Q$.
Find the length of $P Q$.

